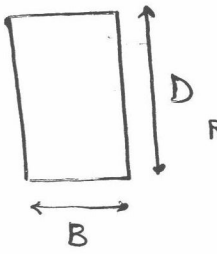
(یافتن ضرایب شکل مقاطع مختلف ν)

* در مقاطع زیر مطلوب است اثبات روابط Z_p و ν

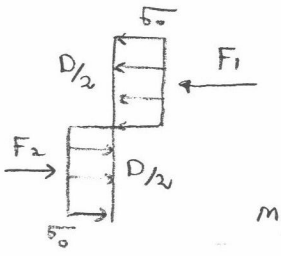
الف) Solid Rectangular



$$Z_p = \frac{1}{4} BD^2$$

$$\nu = 1.50$$

گام اول: یافتن M_p



$$F_1 = F_2 = F = \frac{D}{2} \cdot \sigma_0 \cdot B = \frac{BD\sigma_0}{2}$$

$$M_p = 2F \cdot \frac{D}{2} \cdot \frac{1}{2} = \frac{FD}{2} = \frac{BD^2\sigma_0}{4}$$

$$Z_p = \frac{M_p}{\sigma_0} = \frac{BD^2}{4} \quad \checkmark$$

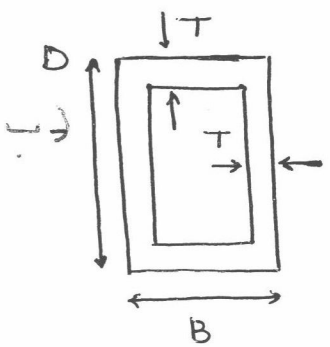
گام دوم: یافتن S_e or M_y

$$S_e = \frac{I}{\frac{D}{2}} = \frac{2I}{D} \quad \text{and} \quad I = \frac{1}{12} BD^3$$

$$S_e = \frac{2}{D} \cdot \frac{1}{12} BD^3 = \frac{BD^2}{6}$$

$$\nu = \frac{Z_p}{S_e} = \frac{BD^2}{4} \cdot \frac{6}{BD^2} = 1.50 \quad \checkmark$$

ب) می‌خواهیم مقدار S_e را یافته و تقسیم آن بر σ_0 بدست آوریم.



$$Z_p = BT(D-T) + \frac{T}{2}(D-2T)^2$$

if $B = D$

$T = 0.050D$ $\rightarrow \nu = 1.18$

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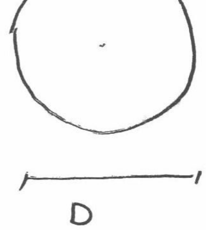
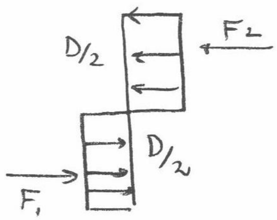


$$\nu = \frac{16}{3\pi} = 1.70$$

گام 1) : یافتن Mp



$$F_1 = F_2 = F = \frac{1}{2} \cdot \frac{\pi}{4} D^2 \cdot \sigma_0$$

$$= \frac{\pi D^2 \sigma_0}{8}$$

محل اثر نیروها : $\frac{4r}{3\pi} = \frac{4}{3\pi} \cdot \frac{D}{2} = \frac{2D}{3\pi}$

$$M_p = 2F \cdot \frac{2D}{3\pi} = \frac{4D}{3\pi} \cdot \frac{\pi D^2 \sigma_0}{8} = \frac{D^3 \sigma_0}{6}$$

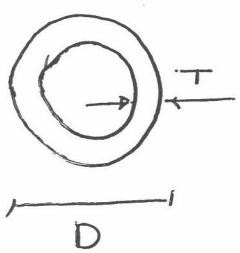
$$Z_p = \frac{M_p}{\sigma_0} = \frac{D^3}{6} \quad ✓$$

گام 2) : یافتن S_e که :

$S_e = \frac{\pi}{32} D^3$ برای مقطع توپر دایره‌ای

$$\nu = \frac{Z_P}{S_e} = \frac{D^3}{6} \cdot \frac{32}{\pi D^3} = \frac{16}{3\pi} \quad ✓$$

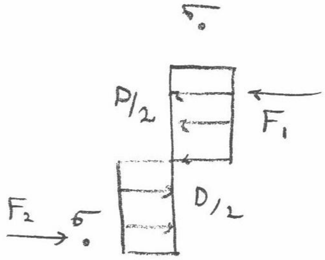
د)



$$Z_p = \frac{1}{6} D^3 \left(1 - \left(1 - \frac{2T}{D}\right)^3\right)$$

if $T = 0.050 D$ $\quad \nu = 1.34$

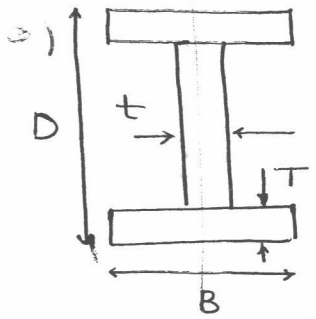
$T \ll D$ $\quad \nu = \frac{4}{\pi} = 1.27$



گام 1) : یافتن Mp

$$F_1 = F_2 = \frac{1}{2} \frac{\pi}{4} \left(D^2 - (D-2T)^2\right) \sigma_0$$

$$= \frac{\pi \sigma_0}{8} \left(D^2 - (D-2T)^2\right)$$

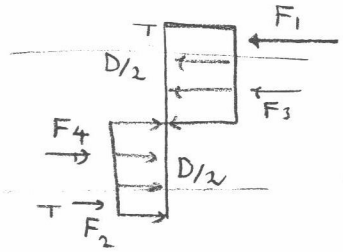


Axis X – X

$$Z_P = BT.(D-T) + \frac{1}{4} t (D-2T)^2 \qquad \nu \cong 1.14$$

Axis Y-Y

$$Z_P = \frac{1}{2} TB^2 + \frac{1}{4} (D-2T) t^2 \qquad \nu \cong 1.60$$



الف – Axis X-X : $M_P = ?$

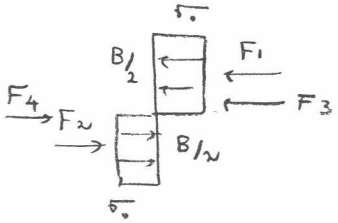
$$F_1 = F_2 = BT.\sigma_0$$

$$F_3 = F_4 = \left(\frac{D}{2} - T\right).t.\sigma_0 = \frac{\sigma_0 t}{2}(D-2T)$$

$$M_P = 2F_1.\left(\frac{D}{2} - \frac{T}{2}\right) + 2F_3.\left(\frac{D}{2} - T\right).\frac{1}{2}$$

$$= BT\sigma_0 (D-T) + \frac{\sigma_0 t}{2}.\frac{1}{2}(D-2T)^2$$

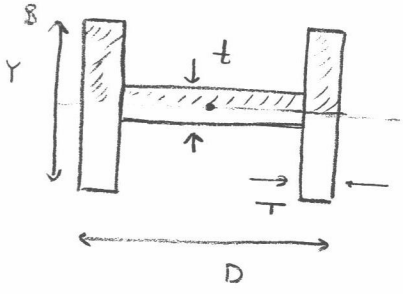
$$Z_P = \frac{M_P}{\sigma_0} = BT.(D-T) + \frac{t}{4}(D-2T)^2 \quad \checkmark$$



ب – Axis Y-Y : $M_P = ?$

$$F_1 = F_2 = 2.\frac{B}{2}.T.\sigma_0 = BT\sigma_0$$

$$F_3 = F_4 = (D-2T).\frac{t}{2}.\sigma_0 = \frac{\sigma_0 t}{2}(D-2T)$$



$$M_{P_1} = 2F_1.\frac{B}{4} = \frac{BF_1}{2} = \frac{B^2 T\sigma_0}{2}$$

$$M_{P_2} = 2F_3.\frac{t}{4} = \frac{F_3 t}{2} = \frac{\sigma_0 t^2}{4}.(D-2T)$$

$$M_P = M_{P_1} + M_{P_2}$$

$$Z_P = \frac{M_P}{\sigma_0} = \frac{B^2 T}{2} + \frac{t^2}{4}(D-2T) \quad \checkmark$$